



FINANCING CONSTRAINTS AND CORPORATE GROWTH

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ABSTRACT

This paper analyses the dynamic investment and growth prospects of a financially constrained firm. Three types of financing constraints are examined: internal finance, debt ceiling and exponential interest costs. To study the growth dynamics of firms subject to the above constraints, numerical solutions, for assigned parameter values, are provided using the reverse shooting Runge-Kutta algorithm. The simulation results suggest that the firm's real and financial variables are highly correlated for constrained firms, as the optimal policy of these businesses is to over-invest in capital in the initial years, and then deplete this excess capacity in future periods. This, however, results in slower rates of growth for the constrained firm, and for entities facing a debt ceiling, greater rates of fluctuation in their rates of expansion.

KEYWORDS: Financing constraints, corporate growth

JEL CLASSIFICATION: D9, C6, G32

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I. INTRODUCTION

In many developing countries, the two main sources of finance are retained earnings and credit, primarily from commercial banks. Demirgüç-Kunt and Maksimovic (1996) find that even as the stock markets in these countries develop, internal finance and credit remain the dominant source of investment finance. Moreover, a sudden lack of credit or internal finance can be very costly in terms of investment opportunities foregone, and future growth prospects. Financing constraints can also result in increased economic volatility, since it inhibits the ability of firms to plan with any level of certainty.

Most of the theoretical and empirical studies on financing constraints have limited the scope of study to just investment. Financing constraints, however, can influence a wide variety of firm variables such as growth and profitability (Moore, Craigwell and Maxwell, 2003). Empirical studies in the area also suffer from measurement related problems (Hubbard, 1998): for instance, the shadow cost of capital can only be proxied by Tobin's q under very restrictive assumptions – perfect capital markets – which are unlikely to be applicable to most developing states.

In this study, the authors examine three types of financing constraints: internal finance, debt ceilings and exponential interests costs. These models are formulated within a continuous time optimal control framework. In addition to the mathematical derivations of the investment policy functions, the models are also calibrated and simulated using the reverse shooting fourth-order Runge-Kutta method (see Judd, 1998). By using computational techniques, the authors avoid the measurement bias inherent in most econometric approaches. The approach also allows the examination of the long-run dynamics of constrained businesses relative to that for an unconstrained enterprise.

The simulation results suggest that constrained firms would find it more difficult to maximise the lifetime value of the firm because the optimal strategy requires the firm to over-invest. This dynamic path, however, is characterised by slower rates of growth and lower levels of sales/output, and in the case of the debt ceiling, greater volatility.

The paper is organised as follows. Section II introduces the models used while Section III gives the simulated growth paths of firms subject to various types of financing constraints. Section IV concludes.

Consider a small open economy with constant population and an exogenously determined interest rate (r). The representative firm is assumed to produce final output, Y , using the variable inputs real capital, K and labour, L , with constant returns to technology, represented by $Y = F(K, L)$, that is $f_{KK}, f_{LL} < 0, f_{KL} > 0, f_L(K, 0) = \infty$.

A. The Firm's Value Maximisation Problem

Capital is assumed to depreciate by the rate δ , while investment expenditure of I_t , only yields $\phi(I_t, K_t) < I_t$ of new capital due to installation costs. The evolution of the capital stock can therefore be represented as:

$$\dot{K}_t = \phi(I_t, K_t) - \delta K_t \quad (1)$$

where $\phi(I_t, K_t)$ has the following properties, $\phi_{KK}, \phi_{II}, \phi_{KI} < 0, \phi(0, K) = 0 \forall K$. The use of scale dependent costs of adjustment of capital agrees with Goolsbee and Gross (1997) empirical finding of significant non-convexities in adjustment costs.

The firm chooses investment and labour to maximise its discounted stream of future profits (Π):

$$\max \int_0^{\infty} e^{-\rho t} [F(K_t, L_t) - wL_t - I_t] dt \quad (2)$$

where $\rho > 0$ is the rate of time preference and w is the wage rate.

The first order conditions² for this problem yield the now familiar investment expression:

² The first order conditions (FOC) are given by

$$\begin{aligned} [F_L(K_t, L_t) - w] &= 0 \\ 1 + \eta &= \phi_I(I_t, K_t) q_t \\ \dot{q}_t &= q_t [\rho + \delta - \phi_K(I_t, K_t)] - f_K(K_t, L_t) (1 + \eta) \\ q^* \eta &\geq 0, q^* [\phi(I_t, K_t) - \delta K_t] = 0 \text{ and } \eta^* [\Pi_t - I_t] = 0 \\ \lim_{t \rightarrow \infty} e^{-\rho t} q_t K_t &= 0 \end{aligned}$$

$$I_t^u = \frac{1}{\phi_t} \left(\frac{1}{q_t} \right) K_t \quad (3)$$

with $\frac{1}{\phi_t} > 0$ and shows that investment is positively related to the shadow cost of capital.

B. A Firm Constrained by Internal Finance

Assume that the firm's investment is limited, since it cannot issue debt on the local financial market and therefore has to depend on internal finance/retained earnings. This constraint is represented mathematically as:

$$I_t < \Pi_t \quad (4)$$

There could be many reasons why the firm may not be able to borrow on the domestic financial market, such as capital market imperfections and the limited development of the financial market. The internal finance constraint is therefore a reduced form representation of imperfections that are not explicitly modelled in this study.

There are two types of firms in the economy; those that are constrained by internal finance (denoted by superscript c) and those that are unconstrained (represented by superscript u). The manager of the constrained firm maximises Equation (2) subject to Equations (1) and (4), while the director of the unconstrained firm simply maximises Equation (2) subject to Equation (1). The firm's state variable is its stock of equity, where the corresponding co-state variable is denoted by q_t , which can be interpreted as the shadow cost of new capital. For the financially constrained firm, the problem also includes η , the shadow cost of the liquidity constraint. The conditions necessary for the maximisation of the current value Hamiltonian are:

$$[F_L(K_t, L_t) - w] = 0 \quad (5)$$

$$1 + \eta = \phi_t(I_t, K_t)q_t \quad (6)$$

$$\dot{q}_t = q_t[\rho + \delta - \phi_K(I_t, K_t)] - f_K(K_t, L_t)(1 + \eta) \quad (7)$$

$$q^* \eta \geq 0, q^* [\phi(I_t, K_t) - \delta K_t] = 0 \text{ and } \eta^* [\Pi_t - I_t] = 0 \quad (8)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_t K_t = 0 \quad (9)$$

while noting that for the unconstrained firm $\eta = 0$.

Normalising labour to unity, the optimal capital spending functions can be obtained from Equation (6). For the unconstrained firm, the optimal level of investment is given by:

$$I_t^u = \frac{1}{\phi_t} \left(\frac{1}{q_t} \right) K_t \quad (10)$$

while that for the constrained firm is:

$$I_t^c = \frac{1}{\phi_t} \left(\frac{1 + \eta_t}{q_t - \eta_t} \right) K_t \quad (11)$$

noting that $\frac{1}{\phi_t} > 0$. Equations (10) and (11) imply that investment is positively related to the shadow cost of capital, while Equation (11) shows that investment for the constrained firm is negatively correlated with the shadow cost of the liquidity constraint.

C. A Firm Constrained by a Debt Ceiling

In the model presented in Section II-B firms were not allowed to issue debt/obtain loans from financial institutions. However, most businesses usually utilise some proportion of external borrowing, albeit significantly smaller than retained earnings, to finance investment (Oliner and Rudebusch, 1996). To incorporate borrowing into the model, an additional state variable is employed: debt, denoted by B_t . The interest rate on debt, r , is assumed to be fixed overtime.

The evolution of debt can therefore be given by:

$$\dot{B}_t = rB_t - F(K_t, L_t) + I_t - wL_t \quad (12)$$

Since debt is used to finance investment, the change in debt is positively associated to investment, while the greater the profitability of the firm, the smaller the amount of new debt it needs to contract or the greater its ability to repay outstanding debts. The firm, however, encounters a debt ceiling of the form:

$$B_t \leq \bar{B} \quad (13)$$

which may be imposed by the companies articles of incorporation or market driven.

The firm again chooses investment and labour to maximise its discounted stream of future profits:

$$\max \int_0^{\infty} e^{-\rho t} [F(K_t, L_t) - wL_t - I_t - rB_t] \quad (14)$$

The value function presented above differs from that given in Equation (2) as a result of borrowing costs.

The inter-temporal maximisation problem presented involves a state-space constraint. Thus, the state variable is no longer continuous and can experience jumps around so-called junction points, or where the state-space constraint becomes binding. To solve the problem, the study follows the approach proposed by Hestenes (1966), which introduces another constraint that explicitly takes into account the junction points. The conditions necessary for the maximisation of the current value Hamiltonian formed from Equations (13), (12), (11) and (1) are as follows:

$$[F_L(K_t, L_t) - w][1 - \lambda_t - \theta_t] = 0 \quad (15)$$

$$(q_t + \lambda_t - \theta_t)\phi_t(I_t, K_t) = 1 \quad (16)$$

$$B_t \geq 0, \quad \theta_t B_t = 0$$

$$\dot{\theta}_t \leq 0 \quad [= 0 \text{ when } B_t < \bar{B}]$$

$$\dot{q}_t = q[\rho - \phi_k(I_t, K_t) + \delta] - F_k(K_t, L_t)[1 - \lambda_t + \theta_t] \quad (17)$$

$$\dot{\lambda}_t = \rho\lambda_t + r[1 - \lambda_t + \theta_t] \quad (18)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_t K_t = 0 \quad (19)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t B_t = 0 \quad (20)$$

The investment function for the unconstrained firm derived from Equation (15) is:

$$I_t^u = \frac{1}{\phi_t} \left(\frac{1 - \lambda_t}{q_t} \right) K_t \quad (21)$$

noting that $\dot{\theta} = 0$ for the unconstrained firm. However, for the constrained firm, the Kuhn-Tucker conditions require that when $\theta \neq 0$ then $\dot{B} = 0$. From Equation (12), one therefore obtains:

$$I_t^c = F(K_t, L_t) - wL_t - r\bar{B} \quad (22)$$

The constrained firm simply invests output net of labour and borrowing costs. In contrast, the investment function for the unconstrained firm is positively related to the shadow cost of capital and debt, and the capital stock.

D. A Firm Constrained by High Interest Rates

In the model outlined in Section II-C, the firm faced a fixed interest rate at all levels of debt. In practice, however, firms usually pay differing rates of interest depending on the level of debt that they currently hold. To incorporate this phenomenon into the model presented in Section II-C, it is assumed that the firm has exponential debt costs:

$$r_t = \bar{r} \exp(aB_t) \quad (23)$$

Equation (23) implies that as the firm's level of debt rises, so to does the rate of interest it has to pay.

The firm's maximisation problem is the same as that presented in the previous section, abstracting from the debt ceiling. The conditions necessary for the maximisation of the current value Hamiltonian formed from Equations (13), (22), (11) and (1) are as follows:

$$[F_L(K_t, L_t) - w] = 0 \quad (24)$$

$$q_t \phi_t(I_t, K_t) = 1 - \lambda_t \quad (25)$$

$$\dot{q}_t = q_t[\rho + \delta - \phi_k(I_t, K_t)] - F_k(K_t, L_t)(1 - \lambda_t) \quad (26)$$

$$\dot{\lambda}_t = \rho\lambda_t + [\bar{r} \exp(aB_t) + aB_t \bar{r} \exp(aB_t)](1 - \lambda_t) \quad (27)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_t K_t = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t B_t = 0 \quad (28)$$

$$q^*, \lambda^* \geq 0, \quad q^*[\phi(I_t, K_t) - \delta K_t] = 0 \text{ and } \lambda^*[r_t B_t - F(K_t, L_t) + \phi(I_t, K_t) - wL_t] = 0 \quad (29)$$

The investment function for firms that face a constant interest rate and a exponential debt cost is given in Equation (21). Thus, although the rate of interest and therefore the level of investment may differ between firms, the firms' optimal investment curves are similarly sloped.

III. NUMERICAL EXAMPLES

To study the growth dynamics of the firms given above, numerical solutions for assigned parameter values are provided using the reverse shooting Runge-Kutta algorithm (see Appendix). In the numerical solutions that follow, the following benchmark parameter values are used: $\alpha = 0.39$, $\delta = 0.07$, $L = 1$, $\rho = 0.04$, $w = 1$, $c = 0.60$, $\eta^c = 0.50$ and $\eta^u = 0.00$ and $k_T = 2$. The capital share (α) is set to a figure consistent with the findings in growth accounting and comparable calibration studies (Strulik, 1999). Given that the authors are mainly

interested in corporate development or growth (as opposed to absolute values) the variables L and w are set to unity. The rate of depreciation is the average rate of depreciation used by firms in Barbados (see Moore, Craigwell and Maxwell, 2003) and is in line with other studies in the area (Xie and Yuen, 2002). The riskless rate is equal to the time preference rate and is the same as that used by Strulik (1999), one percentage point higher than Xie and Yuen (2002) and two percentage points more than Pratap (2000). The remaining values are chosen arbitrarily, since no similar studies were found which looked at these variables. To ensure that the results were not significantly affected by the choice of these parameters, various values were tried in the simulation phase of the study, but the results were similar and are not reported.

A. A Firm Constrained by Internal Finance

Figure 1 plots the capital stock, growth, shadow cost of capital and investment-capital ratios for both types of corporations (constrained and unconstrained). The solid lines show the optimal paths for the constrained firm, while the broken lines represent those of the unconstrained enterprise.

The capital stock path of the constrained firm is greater than that of the unconstrained enterprise for most of the simulated period, but converges over time. This situation occurs since the optimal strategy for a constrained firm requires that it initially over invests in order to ensure that it has enough capital to maximise its value overtime, and then use its spare capacity overtime by reducing its investment intensity. As the shadow cost of the liquidity constraint rises (η), the greater the level of investment in the initial years and the larger the distance between the capital stock for the two types of corporations. Therefore, the optimal capital path of the constrained firm is more difficult to attain.

As a result of the constraint on investment, the investment intensity of the enterprise, measured by the investment-capital ratio, is higher for the unconstrained firm than that for the constrained entity. This occurs since the shadow cost of an additional unit of capital is greater for constrained entities. The constrained firm finds it optimal to have a lower investment ratio, since it can utilise some proportion of the capital it has from the previous period.

Figure 1 also shows that the optimal growth path for constrained businesses is lower than that for their unconstrained peers. The difference in growth rates is particularly large during the initially years of operation. This finding implies that financing constraints can have a significant impact on a firm's probability of survival, since in the early years of development the firm's

growth rate tends to be lower if it is constrained by internal finance. This finding shows how important it is to have a vibrant market for venture capital finance, and could possibly explain why in some developing countries many young corporations tend to fail (Bartelsman, Scarpetta and Schivardi, 2003).

The implied correlation between growth, g , and the investment-capital ratio and cash and the investment-capital ratio are shown in Figure 2. The figure shows that for the constrained firm there is a strong positive linear association between investment and growth, while this is not the case for the unconstrained entity. The main reason for this outcome, as is shown in the two bottom panels, is that the growth prospects of a constrained firm is highly related to the availability of retained earnings, while this is not the case for the unconstrained firm. The high correlation between cash and growth, however, implies that the volatility of the constrained firm is much greater, since its investment prospects, and by extension its growth path, is tied to the level of cash it has available.

B. A Firm Constrained by A Debt Ceiling

Figure 3 plots the capital stock, growth, shadow cost of capital, investment-capital ratios and debt for the constrained and unconstrained firms. The simulation is undertaken assuming that the firm faces the debt ceiling from its first period of operation. As a result, the firm therefore plans its activities and where it wants to be at period t with full knowledge that the debt ceiling will be in effect. The debt ceiling is set at 0.2, and the firm's debt balance at period T is assumed to be zero.

Similar to the firm constrained by internal finance, the optimal capital stock for the firm when the debt ceiling is binding is higher than for the unconstrained enterprise. The optimal capital stock for both the constrained and unconstrained businesses converge overtime, since the level of debt needed to finance investment eventually declines. Therefore, as the debt ceiling is no longer binding, the dynamics for both types of companies converge.

Figure 3 also shows the growth path for both entities. The dynamics of the unconstrained firm naturally declines overtime. In contrast, the growth path for the constrained entity exhibits a high degree of variance whilst the debt ceiling is binding, and then jumps to the smooth growth path once the constraint is no longer binding. Note also that the rate of growth of the unconstrained firm is everywhere above that for the constrained entity once the constraint is binding. A similar pattern is evident for the investment-capital ratio, which implies that

constrained firms are more likely to exhibit a greater variance, in terms of growth, than for their unconstrained peers.

The debt path of the constrained firm, as expected, lies below that of the unconstrained firm. This implies that borrowing for new investment has to be financed partially from retained earnings and borrowings. One interesting finding, however, is that the firm does not fully utilise its available credit limit, in this scenario 0.2. Firms instead prefer to use internal funds rather than the other proportion of the available credit line in an attempt to guard against overshooting the optimal level of borrowing, and by extension the capital stock, once the constraint is no longer binding. Pratap (2000) reports a similar finding.

Figure 4 shows the correlation between investment, growth and cash. Both types of firms exhibit a high correlation between the investment-capital ratio and growth. However, there is a much higher correlation for the constrained firm, reflected by the slope of the x-y line. This occurs since growth usually results in greater amounts of cash, which for the constrained enterprise implies that investment can expand. In contrast, the unconstrained firm is less restricted by cash and therefore exhibits a lower correlation between growth and investment. The jagged portion of the normally smooth x-y line also reflects the sensitivity between cash and investment.

C. A Firm Constrained by Interest Rates

The simulation for the firm facing exponential interest costs is reported in Figure 5 and 6. The simulation is undertaken assuming that the unconstrained firm encounters a constant borrowing cost of 4%, while \bar{r} in Equation (22) is set at 4% as well. Similar to the previous simulations, the optimal capital stock path for the restricted enterprise lies above that for the constrained business. Moreover, the larger the difference between r and \bar{r} , the greater the distance between the capital stock paths for the two types of firms.

As a result of over investment in the initial years, the restricted firm therefore invests less than its unconstrained peer and obtains a slower rate of growth. Note, however, that the impact on growth is not as dramatic as in the case of the firm restricted by internal finance and a debt ceiling. The figure also shows that the shadow cost of new debt, λ , is much lower than that for the unrestricted firm given that borrowing costs are higher. Therefore, while new borrowing might increase the firm's value by contributing to higher investment, the higher borrowing cost

also have to be taken into account. In addition, since the optimal strategy for the firm calls for it to over-invest, it still has to contract a higher level of debt, despite the greater borrowing costs.

Figure 6 plots the correlation between investment, growth and cash. Similar to the case of a firm with a debt ceiling, there exists a greater correlation between growth and investment for the constrained firm on accounts of the strong relation between investment and cash for the restricted firm.

IV. CONCLUSIONS

This paper provides three models of investment under financing constraints. The models examined allow for a firm to be constrained by internal finance, a debt ceiling or exponential borrowing costs. The simulated results show that the firm's real and financial variables are highly correlated for constrained firms. One key finding of the paper is that liquidity constrained firms optimal policy is to over-invest in capital in the initial years of operation, and then deplete this excess capacity in future periods. This, however, results in slower rates of growth for the constrained firm. Another finding of the study is that firms facing a debt ceiling, usually experience greater rates of fluctuation in their rates of expansion.

The findings of this study are particularly important for developing countries with relatively undeveloped capital markets, since firms in these countries are usually highly dependent on internal finance and debt. However, the results presented in this study show that financing constraints can push the economy onto a lower growth path. This implies that reducing the financing constraints that firms encounter in these countries should be a priority, not only in terms of supply but interest costs as well.

REFERENCES

- Bartelsman, E., Scarpetta, S. and Schivardi, F. (2003). Comparative Analysis of Firm Demographics and Survival: Micro-level Evidence for the OECD Countries. Economics Department Working Paper No. 348, OECD, Paris.
- Demirgüç-Kunt, A. and V. Maksimovic (1996). Stock Market Development and Financing Choices of Firms. *World Bank Economic Review* 10 (2): 223-239.
- Goolsbee, A. and Gross, D. (1997). Estimating Adjustment Costs with Data on Heterogeneous Capital Goods. University of Chicago Working Paper, M.A.
- Hestenes, M.R. (1966). *Calculus of Variations and Optimal Control Theory*, Wiley, New York.
- Hubbard, G.R. (1998). Capital-Market Imperfections and Investment. *Journal of Economic Literature* 36 (1): 193-225.
- Judd, K.L. (1998). *Numerical Methods in Economics*. MIT Press, Cambridge, M.A.
- Moore, W., Craigwell, R. and C. Maxwell (2003). Financing Preferences and Firm Growth in a Small Open Economy. Central Bank of Barbados, Bridgetown.
- Oliner, S.D. and G. Rudebusch (1996). Is There a Broad Credit Channel for Monetary Policy? *Federal Reserve Bank San Francisco Economic Review* 1: 3-13.
- Pratap, S. Do Adjustment Costs Explain Investment-Cash Flow Insensitivity? *Journal of Economic Dynamics and Control* 27 (11-12): 1993-2006.
- Strulik, H. (1999). A Financial Theory of Firm Growth. Cambridge University Working Paper, UK.
- Xie, D. and C. Yuen (2002). A Dynamic General Equilibrium Framework of Investment with Financing Constraint.

APPENDIX: SIMULATING INFINITE-HORIZON DYNAMIC MODELS

The Runge-Kutta method is an approach developed by German mathematicians C.D.T. Runge and M.W. Kutta to solve ordinary differential equations (see Judd, 1998, for a detailed exposition). The technique works by using a formula to look where the solution is going, and then implement a correction when it is needed. The mathematical formulation for the fourth order Runge-Kutta algorithm is as follows:

$$z_1 = f(x_i, Y_i)$$

$$z_2 = f(x_i + \frac{1}{2}h, Y_i + \frac{1}{2}hz_1)$$

$$z_3 = f(x_i + \frac{1}{2}h, Y_i + \frac{1}{2}hz_2)$$

$$z_4 = f(x_i + h, Y_i + hz_3)$$

$$Y_{i+1} = Y_i + \frac{h}{6}[z_1 + 2z_2 + 2z_3 + z_4]$$

where x is the control variable, Y the state variable and h the step size.

For infinite-horizon problems, where the differential equation has to be integrated over long periods, it is preferable to use reverse shooting, since the solution path tends to be sensitive to the starting values chosen. With reverse shooting, one chooses the terminal values and then work back to find the initial state. To reverse time the system of equations is multiplied by -1 and simulated using the equations given above. After the solution path is obtained, the solution sequences are inverted to provide the forward-looking time path for the original system.

FIGURE 1:
Simulation Results – Internal Finance

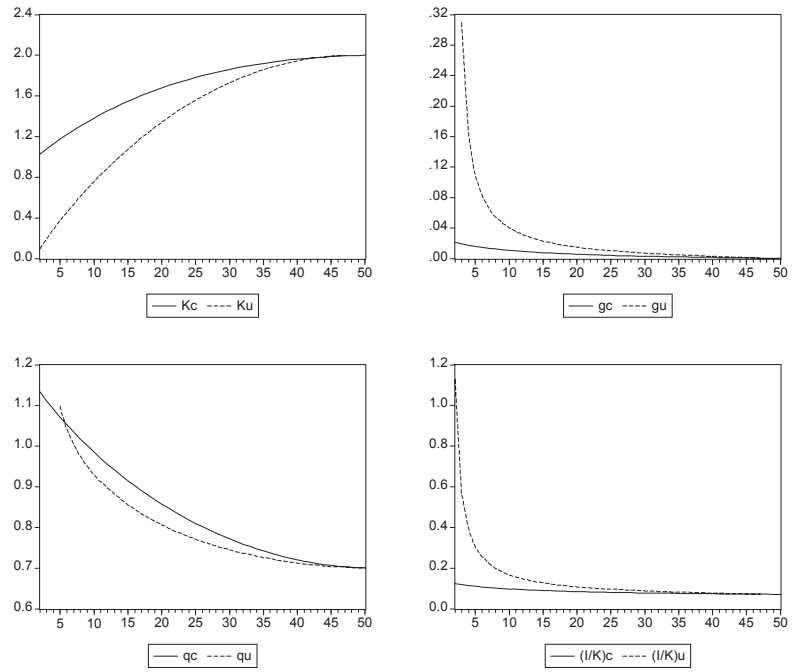


FIGURE 2:
Correlation Between Investment and Cash with/without Internal Finance Constraint

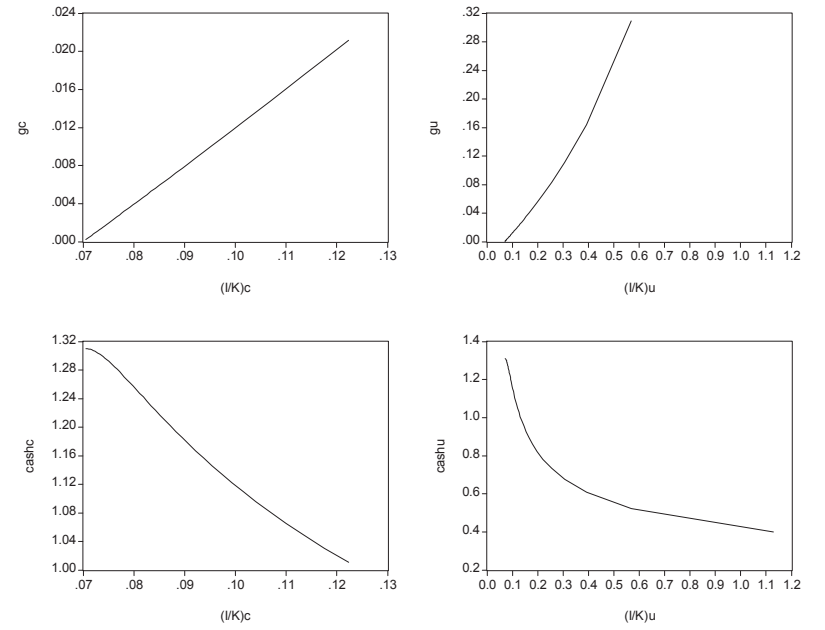


FIGURE 3:
Simulation Results – Debt Ceiling

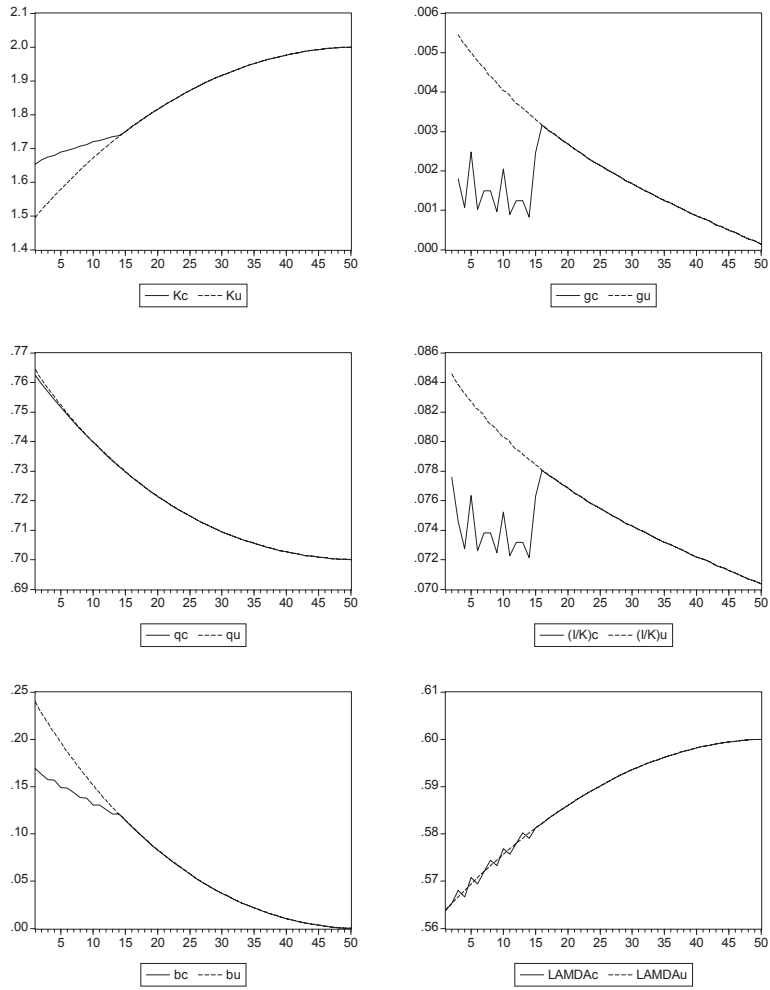


FIGURE 4:
Correlation Between Investment and Cash with/without Debt Ceiling

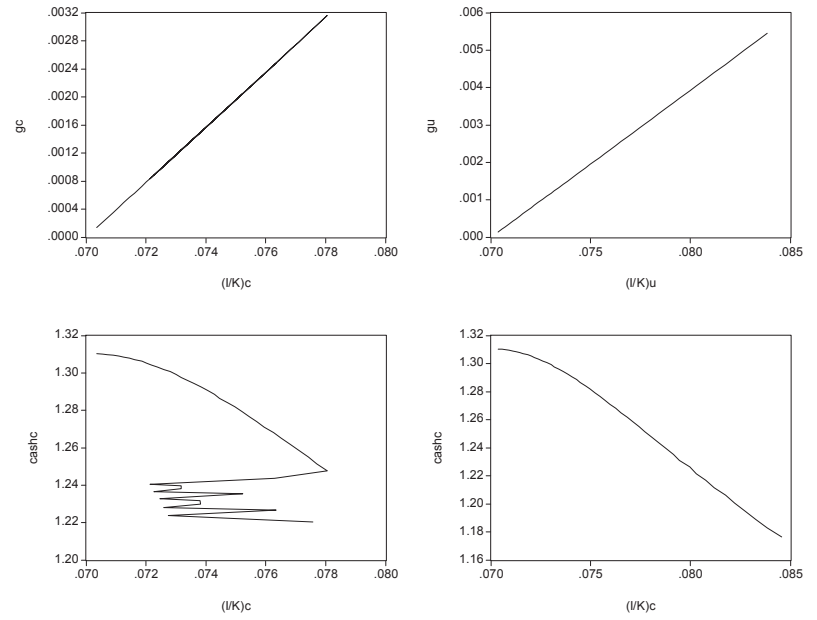


FIGURE 5:
Simulation Results – Exponential Interest Rates

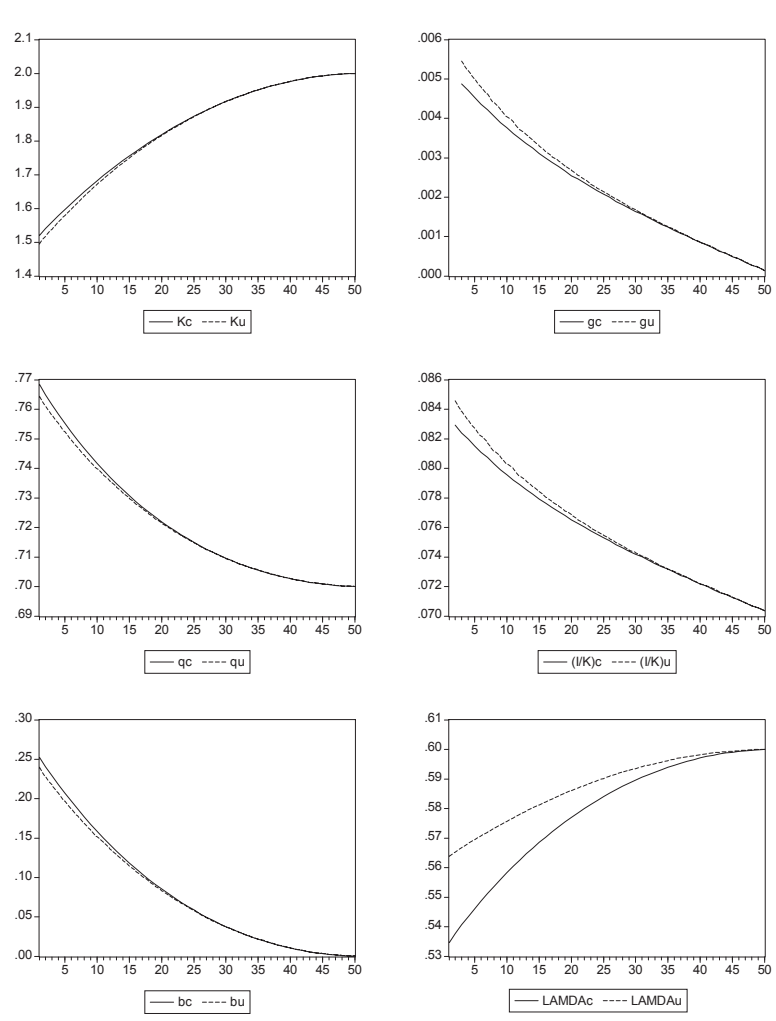


FIGURE 6:
Correlation Between Investment and Cash with/without Exponential Interest Rates

