

# The tail behavior of stock index return on the Jamaican Stock Exchange.

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## **Abstract**

This paper is concerned with the application of extreme value theory (EVT) to daily stock market closing prices on the Jamaican Stock Exchange to determine whether or not stock market returns follow a heavy-tail stable distribution. Our empirical result does not reject a heavy tail stable distribution for returns. It also establishes that the Jamaican Stock Exchange return index has a significantly fatter tail than returns from industrial markets.

**Proposed running head:** Extreme market return.

## I. Introduction

This paper is concerned with the use of extreme value theory (EVT) in modelling the tails of heavy tailed stock index returns distribution on the Jamaican Stock Exchange (JSE). The examination of the stochastic behavior of stock prices on the JSE is a topical issue which has resulted in a number of papers (Agbeyegbe (1994), Hamilton (1998), Jackson (1986), Kim & Brian Langrin (1998), Kitchen (1986), Koot, Miles & Heitmann, (1989), Leon (1996). and Bailey (2002) In all these papers the focus is not on estimating the tail of the distribution of asset returns but on studying the whole distribution. Agbeyegbe (1994) applies the methodology of Porterba & Summers (1988) to look for evidence of mean reversion in stock prices. Hamilton (1998), Kim & Brian Langrin (1998) and Leon (1996) apply the GARCH framework. Jackson (1986), Kitchen (1986) and Koot, Miles & Heitmann, (1989) perform test of randomness to investigate weak form efficiency test of Fama (1965). Bailey (2002) utilizes vector autoregression methodology to assess the effects of monetary and fiscal policy on the behavior of Jamaican stock prices. In this paper we focus on risk management issues. Our modelling strategy is based on EVT, a theory with initial application in hydrology and climatology, see Smith (1989) and the reference therein. <sup>1</sup>

Extreme value theory is concerned with the modelling of extreme events and in the last decade several authors have noted its relevant to the modelling of extreme price movements. It should be noted that though several authors have investigated the statistical distribution of returns and have concluded that returns are "fat tailed", an overall fit of such processes to historical data may not provide an adequate framework for analyzing extreme events, instead, it is suggested that one should use EVT methodology. In particular, the peaks over threshold or POT model is a method for modelling extreme price movements. In the POT model, excess(large) price movements over high thresholds are modelled with generalized Pareto distribution (GPD), this distribution arises in one of the key limit theorems in EVT and do not depend on whether the underlying distribution is normal or fat tailed.

The notion that financial returns are fat tailed is not new and is an outgrowth of several studies devoted to the stochastic behavior of security prices. The development of the first serious study of the stochastic behavior of security prices is often attributed to Bachelier (1900), who in his 1900 dissertation suggested that security prices in successive periods

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<sup>1</sup>A well known reference on EVT modelling in finance is Embrechts, Klüppelberg, & Mikosch (1997).

follow a random process that is best described by a Wiener process. Wiener processes are characterized by independence and normality. Although the "Gaussian" hypothesis was formulated by Bachelier, subsequent empirical work has produced evidence suggesting that the hypothesis is only a good first approximation to describing return distribution. The observed typical empirical distribution of security returns is often leptokurtic, that is "peaked" and "fat-tailed" relative to the normal, and frequently positively skewed. This result was first pointed out by Mandelbrot (1963) and shortly after by Fama (1965), who demonstrated that symmetric stable infinite variance densities were better for describing returns than the Gaussian. It should be noted that subsequent research has emphasized that security returns are best described as mixture models. Campbell, Lo and MacKinlay (1997) discuss some distributional issues.

Much of previous literature focussed on whether actual security prices are indeed generated by a Gaussian process. This is important, since the form of the generating process has important implications for option pricing models and the generalized capital asset pricing model. The recent literature, however, focuses on methods for making inferences relating to extreme movements. The notion of extreme movements in security prices is implicit in current risk management practices and extremal theory that has recently gained popularity in economics for obtaining risk management calculations.<sup>2</sup> Despite several applications of EVT to risk management calculations in several areas, foreign exchange, interest rates and stock markets, for developed and developing countries, relatively little is known about the dynamics of stock returns associated with extreme up and down movements on the JSE. Ours, to our knowledge, is the first attempt at applying EVT methodology to historical data on the JSE.

The organization of the article is as follows: Section 2 introduces two important results in extreme value theory. Section 3 presents our analysis. The conclusions are given in Section 4.

## II. Extreme Value Theory: Some essential results

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<sup>2</sup>An extensive review of conventional risk management modelling can be found in Dowd (1998). Diebold & Santomera (1999), Christoffersen & Errunza (2000) and Neftci (2000) point out the problems in using a normality assumption. An assumption which is commonly adopted in conventional risk measurement methodologies. It is often asserted that, traditional value at risk methodologies are not well designed to cope with extreme movements that take place during financial crisis. Danielsson (2000) points out that statistical analysis made in times of stability do not provide valid inferences in times of instability.

This section states two key results from EVT.<sup>3</sup> The first result concerns the asymptotic distribution of a series of maxima (minima) and it states that under certain conditions, the distribution of the standardized maximum (minima) of the series converges to one of three known distributions: Gumbel, Frechet, or Weibull. A standard form of these three distributions is the generalized extreme value (GEV) distribution. The second results deals with the distribution of excess over a given threshold. The results states that regardless of the underlying distribution of the series under investigation, the limiting distribution of excesses over a high threshold is a generalized Pareto distribution (GPD).

### A The GEV distribution (Fisher-Tippett, Gnedenko result)

Let  $(Y_1, \dots, Y_n)$  be a sequence of independent and identically (iid) random variables from an unknown distribution  $F$  with the maximum  $M = \text{Max}(Y_1, \dots, Y_n)$ . Then there exist normalizing constants  $a_n$  and  $b_n$  such that the limiting distribution  $P[a_n M + b_n \leq y]$  converges in law (weakly) to the following distribution:

$$H_\phi(y) = \begin{cases} \exp(-(1 + \phi y)^{-1/\phi}) & \text{if } \phi \neq 0 \\ \exp(-e^{-y}) & \text{otherwise} \end{cases}$$

while  $1 + \phi y > 0$ .

In this formalization  $\phi$  corresponds to a shape parameter and it indicates the thickness of the tail of the distribution. The tail index  $\alpha = 1/\phi$ . The tail index can be used to assess whether or not moments of the distribution in question exists. This is of practical importance since many results in the field of portfolio theory and derivative pricing rely on the existence of several moments. It should also be noted that estimates of the tail index aid in the computation of probability of extreme events. Lastly, the exact shape of the extreme distributions depend on the sign and size of the tail index.<sup>4</sup> For positive  $\phi$ , the smaller the tail index, the larger the weight of the tail. When the  $\phi$  is equal to zero, the distribution  $H$  corresponds to a Gumbel type. When  $\phi < 0$  we have a Weibull distribution and when  $\phi > 0$  we have a Frechet distribution. The Frechet distribution corresponds to fat-tailed distributions and has been found to be most appropriate for fat-tailed financial data.

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<sup>3</sup>General texts on EVT include Falk, Hüsler, & Reiss (1994) and Embrechts, Klüppelberg & Mikosch (1997).

<sup>4</sup>Notice that some writers define the tail index as  $\phi$ .

## B The excess beyond a threshold (Pickands, Balkema and de Haan result)

This sub-section states result relating to excesses over a high threshold  $u$ . Once again, let  $(Y_1, \dots, Y_n)$  be a sequence of independent and identically (iid) random variables from an unknown distribution  $F$ . We are interested in excess losses (returns) over a high threshold  $u$ . Let  $y_0$  be the finite or infinite right end point of the distribution  $F$ . The distribution function of the excesses over the threshold  $u$ , is given by

$$F_u(y) = P\{Y - u \leq y \mid Y > u\} = \frac{F(y+u) - F(u)}{1 - F(u)}$$

for  $0 \leq y < y_0 - u$ .  $F_u(y)$  is the probability that a return exceeds the threshold  $u$  by no more than amount  $y$ , given that the threshold is exceeded. The resulting distribution from above is the generalized Pareto distribution (GPD) which is usually expressed as a two parameter distribution with degrees of freedom.

$$G_{\phi, \sigma}(y) = \begin{cases} 1 - (1 + \phi y / \sigma)^{-1/\phi} & \text{if } \phi \neq 0 \\ 1 - \exp(-y/\sigma) & \text{otherwise} \end{cases}$$

where  $\sigma > 0$ , and the support is  $y \geq 0$  when  $\phi \geq 0$  and  $0 \leq y \leq -\sigma/\phi$  when  $\phi < 0$ . The GPD subsumes three other distributions under its parametrization. For example, when  $\phi = 0$ , we obtain a Type I (exponentially declining) distribution. If  $\phi < 0$ , we have a Type II (power declining), the usual Pareto, distribution. If  $\phi > 0$ , we obtain a Type III (constant declining) distribution. For type I and III distributions all moments exist. In the case of the Type II limiting distribution, only moments up to the integer part of  $\alpha$  exist and higher moments are infinite. That is to say, that higher moments do not exist, because the tails do not decay rapidly enough when 'weighted' by the tail probability to be integrable.

The class of distributions where the tail decays like an exponential function is quite large and it includes the normal distribution, discrete mixtures of normal and mixed diffusion processes. For all these Type I distributions, all moments exist. Just as in the case of Type I distributions, the class of distributions where the tail decays like a power function is also large. These Type II distributions include the Pareto, Burr, loggamma, Cauchy, t-distributions and ARCH processes as well as various mixture models.

Given these three types, our task in this paper will be to uncover which type best describes the extremes of stock returns on the JSE. Our prior belief is that the returns will be best

captured by a Type II distribution. It is well known that one can rule out Type I distribution for stock returns because they have thin tails. One can also rule out Type III distributions, since the distribution specifies upper bounds for returns when in fact returns are unbounded.

### III. The Data and Empirical Analysis of JSE Stock Index returns

The discussions in the previous section would suggest that when we have data from an unknown underlying distribution, we may be able to successfully approximate the distribution of excesses over sufficiently high thresholds by a generalized Pareto distribution  $G_{\phi,\sigma}(y)$  for some values of  $\phi$  and  $\sigma$ . This is the modelling approach which can be found in recent papers on EVT (Smith 1989, Davison & Smith 1990 among many). In what follows we will attempt to obtain the tail index for returns on the JSE by first fitting the GPD.<sup>5</sup> By estimating  $\phi$  and  $\sigma$  from the GPD it is relatively easy to compute the tail index  $\alpha$  since  $\alpha = 1/\phi$ .

#### A Data

The data used in this study are obtained from the Jamaican Stock Exchange web site<sup>6</sup> and consist of 2579 daily observations (closing prices) of the JSE Composite Index, Jan. 5, 1988-Dec. 31, 2001. Table 1 in the Appendix provides summary statistics of the level and first difference of the data series. Table 2 in the appendix provides goodness of fit tests for three models. Interestingly, these results do not lead to definite conclusion on the form of the stock returns. Following extreme value theory, we define the extremes as extremes over high thresholds. Specifically, the extreme returns are defined as those more than two standard deviations away from the sample mean of daily index price changes, which corresponds to almost two percent of the right and left tails of the distribution. Table 3 in the appendix displays the mean, standard deviation, maximum and minimum values of the extremes.

#### B Results

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<sup>5</sup>Tail estimates using extreme value theory have been estimated for stock returns by Jansen and de Vries (1991), Lux (2000), Vilasuso and Katz (2000), Susmel (2001), Agbeyegbe (2002) and for exchange rates by Koedijk, Schagans and de Vries (1990) and Hols and de Vries (1991)

<sup>6</sup><http://www.jamstockex.com>

The generalized Pareto distribution can be fitted to data on excesses over high thresholds by different methods including the maximum likelihood method and the method of probability weighted moments (PWM). For the current paper, we choose maximum likelihood method. Hosking & Wallis (1987) provide relative merits of the various methods.

Table 4 presents the maximum likelihood estimates of the parameters of the generalized Pareto distribution (GPD) and the maximized log likelihood values for index return. According to the parameter estimates in Table 4, the estimated parameter of  $\phi$  for the local maxima is 0.404 suggesting a value of the tail index to be 2.475. For the local minima, the estimated value of  $\phi$  is 0.572 giving a tail index value of 1.748. The estimated shape parameter ( $\phi$ ) of the local minima is greater than that for the local maxima. Since the higher the  $\phi$ , the fatter the distribution of the extremes, the distribution of minima changes in stock returns is fatter than that of the maximal changes.

These results for a Caribbean emerging market can be compared to the results in Susmel (2001), which is based on weekly data over the period covering part of 1989 to 1996. Susmel reports that Latin American emerging markets have significantly fatter tails than industrial markets. Our results confirm this observation for Jamaica. <sup>7</sup>

#### IV. Conclusions

We have used extreme value theory to investigate the tail behavior of stock returns from the JSE. We offer the following conclusions. First, for our sample of daily data covering the period Jan. 5, 1988- Dec. 3, 2001, the distribution of minima changes in stock returns is fatter than that of the maximal changes. This indicated that the distributions of the return are far from Gaussian. Second, estimated value of the tail index for the maxima is 2.475 and that for the minima is 1.748. These results suggest that stock returns from the Jamaican Stock Exchange have significantly fatter tails than stock returns from industrial markets.

The work reported here is merely a starting point on the Jamaican Stock Exchange. A wide variety of further questions present themselves for subsequent research. Of particular interest is an analysis of quantile measure of risk and their calculation using EVT. Another topic of relevance is an investigation to uncover whether there is evidence of a 'memory' effect between extreme stock return movements. Finally, an analysis of stock return volatility

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<sup>7</sup>While it might be interesting to examine the case of other Caribbean markets (Barbados together with Trinidad and Tobago), data problems have led us not to focus on them.

based on EVT-GARCH (McNeil and Frey (2000) would be useful. These and other issues are currently being investigated.

**TABLE 1 : Summary Statistics, 01/05/1988 - 12/31/2001**

Variable	Mean	Std. Deviation	Skewness	Kurtosis	Minimum	Maximum
Stock Index level	9.445	0.902	-1.239	0.354	7.167	10.491
Stock index changes	0.001	0.021	-1.182	387.969	-0.586	0.551

Note: Summary statistics are for the log of the levels of the stock index and for the changes in log of the stock index.

**TABLE 2 : Goodness of fit tests for alternative distributions**  
 Jamaican Stock Market Index Return, 01/05/1988 - 12/31/2001

	K-S	C-S
Normal	0.1893 ( $P < .001$ )	2500.078 ( $P < .001$ )
t (5 dof)	.477 ( $P < .001$ )	104764.2 ( $P < .001$ )
t (10 dof)	0.477 ( $P < .001$ )	104161.3 ( $P < .001$ )

Note: This table reports goodness of fit test of the log of index changes for three alternative distributions. Both Kolmogorov-Smirnov (K-S) and the chi-squared (C-S) goodness of fit results are reported. Ritchev (1986) compares both and states the C-S is superior.

**TABLE 3 : Summary Statistics, 01/05/1988 - 12/31/2001**

Variable	n	Mean	Std. Deviation	Minimum	Maximum
Local Maxima Extreme Stock Return	57	0.064	0.009	0.037	0.551
Local Minima Extreme Stock Return	33	-0.067	0.095	-0.586	-0.034

Note: The maxima extreme samples are those greater than 1.65 of the estimated standard deviation of the whole sample.

The minima extreme samples are those less than 1.65 of the estimated standard deviation of the whole sample.

n is the corresponding number of observations above each respective threshold.

**TABLE 4 : Maximum Likelihood Estimates of the Generalized Pareto Distribution, 01/05/1988 - 12/31/2001**

Variable	$\sigma$	$\phi$	Log-likelihood
Local Maxima Extreme Stock Return	0.015	0.404	-160.215
Standard errors	0.003	0.167	
Local Minima Extreme Stock Return	0.012	0.572	-93.740
Standard errors	0.003	0.242	

Note: This table presents the maximum likelihood estimates of the scale ( $\sigma$ ) and shape ( $\phi$ ) parameters of the generalized Pareto distribution.

$$G_{\phi,\sigma}(y) = \begin{cases} 1 - (1 + \phi y/\sigma)^{-1/\phi} & \text{if } \phi \neq 0 \\ 1 - \exp(-y/\sigma) & \text{otherwise} \end{cases}$$

The last column reports the maximized log likelihood value. Threshold for maxima 0.0365 and for minima -0.0341 Estimation was done using S-PLUS with a FinMetrics module. <sup>8</sup>

## References

- Agbeyegbe, T. D., "Some stylized facts about the Jamaica stock market," *Social and Economics Studies*, 43, (1994), pp. 143-156.
- Agbeyegbe, T. D., "The tail index of stock returns in emerging equity markets", (2002), Department of Economics, Hunter College, CUNY.
- Bachelier, L., "Theorie de la speculation," *Annales de l'Ecole Normale Supérieure*, Ser. 3, 17, (1900) 21-84.
- Bailey, K., "Macroeconomic Fluctuations, Economic Policy and the Jamaican Stock Market (1991-1999)" *Social and Economics Studies*, forthcoming (2002).
- Balkema, A & de Haan ., "Residual life time at great age," *Annals of Probability* 2, (1974), 792-804.
- Campbell, J. Y., Lo, A.W., & Criaq MacKinlay, A., *The Econometrics of Financial Markets* (Princeton University Press), (New Jersey, 1997).
- Christoffersen, P.F. & Errunza, V.R., "Towards a global financial architecture: capital mobility and risk management," *Economic Market Review*, 1, (2000), 2-19.
- Danielsson, J., "The emperor has no clothes: Limits to risk modelling," London School of Economics (2000).
- Davison, A., "Modelling excesses over high thresholds, with an application," in : J. de Oliveira, ed., *Statistical Extremes and Applications*, (D. Reidel), (Amsterdam, 1984), 461-482.
- Davison, A. & Smith, R., "Models for exceedances over high thresholds (with discussion)," *Journal of the Royal Statistical Society, Series B*, 52, (1990), 393-442.
- Diebold, F. X. & Santomera, A., "Financial risk management in a volatile global environment," *Asia Risk*, (1999), 35-36.
- Dowd, K., *Beyond Value at Risk: the New Science of Risk Management*, (Wiley) (New York, 1998).
- Embrechts P., Klüppelberg, C. & Mikosch, T., *Modeling Extremal Events for Insurance and Finance*, (Springer Verlag)(New York, 1997).

Falk, M. Hüsler, J., & Reiss, R., *Laws of Small Numbers: Extreme Rare Events*, (Birkhäuser)(Basel, 1994).

Fama, E., "The behavior of stock market prices," *Journal of Business*, 20 (1965), 34-105.

Fisher, R. & Tippett, L., "Limiting forms of the frequency distribution of the largest or smallest member of a sample," *proceedings of the Cambridge Philosophical Society*, 24, (1928), 180-190.

Hamilton, J., "The GARCH and volume relationship with heteroscedasticity in stock returns on the Jamaican Stock Exchange", in :Kim, S. W. & Agbeyegbe, T. D., *Econometric Modelling of Issues in Caribbean Economics and Finance*. (CCMS), (St. Augustine, 1998), 35-65.

Hosking, J. & Wallis, J., "Parameter and quantile estimation for generalized Pareto distribution," *Technometrics*, 29, (1987), 339-349.

Jackson, J., "The stock market in the Caribbean: The Jamaican experience," *Caribbean Finance and Management*, 2, 1, (1986) 61-68

Jansen, D. W., & de Vries, C. G., "On the frequency of large stock returns: putting booms and busts into perspective," *The Review of Economics and Statistics*, 73, (1991) 18-24.

Kim, S. W. & Brian Langrin, R., "Stock price movements and volatility spillovers under foreign exchange liberalization", in :Kim, S. W. & Agbeyegbe, T. D., *Econometric Modelling of Issues in Caribbean Economics and Finance*. (CCMS), (St. Augustine, 1998), 69-99.

Kitchen, R., "The role of the Jamaican Stock Exchange in the capital market," *Caribbean Finance and Management*, 2, 2, (1986) 1-23.

Koedijk, K. G., Schafgans, M. M. A., & de Vries, C. G., "The tail index of exchange rate returns," *Journal of International Economics*, 29, (1990) 93-108.

Koot, R., Miles, J., & Heitmann, G., "Security risk and market efficiency in the Jamaican Stock Exchange," *Caribbean Finance and Management*, 5, 2, (1989) 18-33.

Leon, H., "Volatility persistence on Jamaican stock returns", in :Nicholls, S, Leon, H & Watson, P., *Problems and Challenges in Modelling and Forecasting Caribbean Economies*. (CCMS), (St. Augustine, 1996), 270-295.

Lux, T., "On moment condition failure in German stock returns: an application of recent advances in extreme value statistics," *Empirical Economics*, 25 (2000) 641-652.

Mandelbrot, B., "The variation of certain speculative prices" *Journal of Business*, 36, (1963), 394-419.

McNeil, A. J., "Software for Extreme Values in S-PLUS: EVIS Version 4," Documentation for EVIS software package, (2002).

McNeil, A. J., & Frey, R., "Estimation of tail related risk measures for heteroskedastic financial time series: an extreme value approach," *Journal of Empirical Finance*, 73, (2000) 271-300.

Neftci, S., "Value-at-risk calculations, extreme events, and tail estimations" *Journal of Derivatives*, Spring, (2000), 23-38.

Poterba, J. M & Summers, L. H., "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, (1988), 22, 27-60.

Pickands, J., "Statistical inference using extreme order statistics" *The Annals of Statistics*, 3, (1975),119-131.

Ritchey, R, J., "An application of the Chi-Squared goodness of fit test to discrete common stock returns" *Journal of Business and Economic Statistics*, 4, (1986), 243-254.

Smith , R., "Extreme value analysis of environmental time series: an application to trend detection in ground-level ozone" *Statistical Science*, 4, (1989), 367-393.

Susmel , R., "Extreme observations and diversification in Latin America emerging equity markets" *Journal of International Money and Finance*, 20, (2001), 971-986.

Vilasuso, J., & Katz, D., "Estimation of the likelihood of extreme returns in international stock markets," *Journal of Applied Statistics*, 27, (2000) 119-130.