Some Aspects of External Debt and Economic Growth

by

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Introduction

The genesis of the present liquidity/external debt problems that many developing countries face may be traced in part to the two massive oil price increases of 1973-74, and 1979-80. The oil import bill of the non-oil developing countries escalated sharply, reaching more than $20 billion in 1978, then jumping to more than $50 billion in 1980 and 1981. The effect has been the large and growing deficits on their current accounts. However, the impact of the high oil prices on import bills alone does not provide the full explanation for the present difficulties. The fact is that some of the most heavily indebted countries are net oil exporters. Domestic policies have had a role to play. While borrowing countries have all wished to expand their industrial base to meet the needs of a growing population, the expansion was in most cases occasioned by large budget deficits mostly financed by foreign capital inflows, a higher proportion of which were in the form of project loans. Furthermore, between the period 1974-81, the external debt profile of developing countries took on a new dimension. International banks increasingly became an important source of funds for these countries. As the share of these commercial lenders increased, the debt service burden became heavier.

In the usual sequence of growth and debt stages (De Vries, 1971 in G. Meier, 1976 p. 352) a basic assumption is that income rises in subsequent phases of external debt dependence so that capital mobilized domestically and borrowed abroad is effectively used to increase output. Unfortunately, this is not what usually happens. In some cases, instead of the foreign capital complementing domestic savings, they invariably replaced the latter.

The main focus of this paper is to utilize the growth parameters of a representative country to discuss the question of complementarity of domestic and foreign capital at the theoretical level. In addition, we intend to investigate the so-called critical interest rate above which a country's debt servicing problems become explosive.

Let	Y	=	Gross domestic product (GDP)
	C	=	Consumption expenditure
	S	=	Domestic savings
	I	=	Gross investment
	g	=	Growth rate of GDP
	K	=	Stock of capital
	L	=	Loans disbursed in a period
	i	=	Interest rate on loans
	B	=	Total outstanding debt
	A	=	Amortisation payments
	X	=	Total exports
	M	=	Total imports
	T	=	Total taxes
	k	=	Incremental capital output ratio (ICOR)

By definition,

$$g = \Delta Y/Y \qquad (1)$$

Multiplying the innumerator and denominator of (1) by I, we have

$$g = \frac{\Delta Y}{Y}\,\frac{I}{I}$$

$$\text{or} \quad g = (I/Y)/(\Delta K/\Delta Y) \qquad (2)$$

$$\text{or} \quad g = (I/Y)/k \qquad (3)$$

where $I = \Delta K$ and $k = \Delta K/\Delta Y$

By definition $S = Y - C$

Hence $S = I + X - M$

or $I = S + M - X$

$$\text{Therefore,}\quad g = \left(\frac{S + M - X}{Y}\right)/k \qquad (4)$$

Invoking the balance of payments equilibrium condition

$$M - X = L - iB - A$$

we may re-write (4) as

$$gk = (S/Y) + \left(\frac{L - iB - A}{Y}\right) \qquad (5)$$

The implied assumption here is that all foreign capital inflows are obtained from borrowing. But if additional capital is to be borrowed to supplement domestic capital, the minimum marginal productivity of capital should not be lower than the rate of interest charged on the loan. Thus in (5) we can set k = i, reducing the equation (5) to

$$g = (1/i)[\frac{S}{Y} + \frac{L - iB - A]}{Y}$$

$$\text{or} \quad g = (1/i)[\frac{S + L]}{Y} - \frac{[iB + A]}{Y} \qquad (6)$$

Expression (6) has three familiar implications. Firstly, growth of GDP is greater the smaller the interest rate charged on loans. Secondly, growth is positive only if the ratio to GDP of the sum of domestic savings and gross foreign capital inflows is larger than the sum of amortisation and interest payments as a proportion of GDP. Conversely, the economy will experience negative growth if the ratio to GDP of amortisation and interest payments is larger than domestic savings plus gross foreign capital inflows as a proportion of GDP. The third implication which flows from the second is that no growth is possible if the sum of domestic savings and gross foreign capital inflows exactly matches the sum of amortisation and interest payments.

The critical interest rate for which long-term growth is zero can be determined in (6) by setting g = o, giving the expression

$$i = (S + L - A)/B \qquad (7)$$

It must be stressed that expression (7) holds in the long-run and is applicable to the situation where all domestic savings and foreign capital inflows are utilised to service national debt, leaving no residue for investment.

Expression (6) assumes that the level of foreign loans contracted in a period is not dependent on the interest rate charged on those loans. While this has been the popular practice for borrowing developing countries where the need for foreign capital rather than the interest rate determines the level of loans, it may not strictly be applicable to all other countries. We therefore relax that simple assumption and consider the situation where the level of loans is a function of the interest rate charged.

We may re-write expression (6) as

$$g = (1/i) \ [\frac{S + L(i)]}{Y} - \frac{[iB + A]}{Y} \qquad (8)$$

The implications for growth discussed above for the first case still holds. However, to determine the critical interest rate for which there would be no growth, we differentiate (8) with respect to i.

$$\text{From (8)} \ \frac{dg}{di} = -i^{-2} \ [\frac{S+ L(i)}{Y} - \frac{iB + A}{Y}] + i^{-1} \ [\frac{L'(i) - B}{Y}] \qquad (9)$$

$$\text{where} \quad L'(i) = \frac{\partial L(i)}{\partial i}$$

Thus $dg/di = 0$ implies

$$i^{-2}[(\frac{S + L(i)}{Y} - \frac{iB + A}{Y}] - i^{-1} \ [\frac{L\ (i) - B}{Y}] = 0 \qquad (10)$$

Further manipulations reduces (10) to

$$i(L'(i) - L(i) + A - S = o \qquad (11)$$

Equation (11) is a differential equation in (i). To solve it,

Let L(i) = y, i = x, and A - S = C

Then (11) becomes

$$x \frac{dy}{dx} - \quad + C = 0$$

$$\frac{dy}{dx} - \frac{y}{x} = \frac{-C}{x} \qquad (12)$$

Equation (12) takes the standard form of an ordinary differential equation,

$$\text{i.e.} \quad y' + f(x)y = g(x) \qquad (13)$$

where $f(x) = -1/x$ and $g(x) = \frac{-C}{x}$

Let $F(x) = \int f(x)\,dx = \int 1/x\,dx = - \log x$

The general solution for (13) is of the form

$$y(x) = e^{-F(x)} [G(x) + B] \qquad (14)$$

(See Blaine and Schultze [1973,] p. 202)

where $F(x) = \int f(x)dx = - \int 1/x\,dx = -\log x$

$G(x) = \int g(x)e^{F(x)}\,dx$

and B = a constant of integration.

But $G(x) = \int g(x)e^{F(x)}dx = - C \int (1/x\, e^{-\log x})\,dx$

Hence (14) becomes

$$y(x) = e^{\log x} [- C \int (1/x\, e^{-\log x})\,dx + B] \qquad (15)$$

Let $u = \log x$,

Thus $du = 1/x\,dx$

substituting in (15) gives

$$y(x) = e^{u} (-C \int e^{-u}du + B)$$

$$\text{or} \quad y(x) = e^{u} (Ce^{-u} + B)$$

$$\text{i.e.} \quad y(x) = Be^{u} + C$$

$$\text{or} \quad y(x) = Be^{\log x} + C \qquad (16)$$

To obtain a particular solution for (16) we need to find B the constant of integration. Since log (o) is undefined, the value of B can only be determined where $x \neq o$. For our purposes, we assume an initial average value for $x = \bar{x}$.

Then $y(x) = B_e \log \bar{x} + \bar{C}$

or $B = \frac{1}{e^{\log x}} \; y(\bar{x}) - \bar{C}$ (17)

Therefore the particular solution for (16) takes the form

$$y(x) = e^{-\log \bar{x}} [y(\bar{x}) - \bar{C}]\, e^{\log x} + C \qquad (18)$$

In terms of the original notations, we have

$$L(i) = (e^{-\log \bar{i}} [y(\bar{i}) + \bar{S} - \bar{A}])\, e^{\log i} + A - S \qquad (19)$$

From (19) the critical interest rate will take the form

$$e^{\log i} = \frac{[L(i) + S - A]}{e^{-\log \bar{i}} [L(\bar{i}) + \bar{S} - \bar{A}]} \qquad (20)$$

$$= \frac{e^{\log \bar{i}} [L(i) + S - A]}{[L(\bar{i}) + \bar{S} - \bar{A}]}$$

Therefore $\log i = \log \left[\frac{e^{\log \bar{i}} [L(i) + S - A]}{L(\bar{i}) + \bar{S} - \bar{A}}\right]$

or $$\log i = \log \bar{i} + \log \frac{(L(i) + S - A)}{L(\bar{i}) + \bar{S} - \bar{A})} \qquad (21)$$

One implication from (21) is that in the case where interest rate affects the level of loans, the critical interest rate above which growth would not occur is independent on the external debt levels.

In the next stage of the study an attempt will be made to obtain some numercial results for the model discussed above, using data for Barbados and other developing countries. However, before this can be done, the model should be taken from its present static system to a dynamic one in order to establish a definite time pattern for the analysis. The model, in its present form, can only give quanlitative implications for long-term growth arising from the impact of some of the key economic variables such as the interest rate, the level of new loans and the level of debt outstanding.

References

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