First Draft

THE RELIABILITY OF ALTERNATIVE REAL PRODUCT ESTIMATORS: THE CASE OF MANUFACTURING IN BARBADOS

by

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It is quite rare to have comprehensive data on expenditure, prices and quantity for a given industry. Such a situation exists with the manufacturing sector alone in Barbados. In the sections that follow, this paper will attempt to draw inferences about the relative reliability of the various methods of real product accounting using manufacturing data as a test case.

At the outset it must be realised that the most reliable estimate of real manufacturing product is given by the manufacturing category of the index of industrial production prepared by the statistical department. This particular category will be referred to as the manufacturing index in subsequent discussion. The manufacturing index must be regarded as the most reliable estimate of real manufacturing product because producer price indexes (required for the more popular deflation procedures) do not exist.

The manufacturing index is largely prepared from quantity relatives obtained from annual surveys of industrial firms. In a few cases quantity relatives cannot be obtained and hours worked or employment is used. Weights for the various categories are determined by periodic full scale censuses of value added. The first such census was undertaken for the period 1970-73 and formed the basis of the 1971 manufacturing index. The second census covered the period 1979-80 and formed the basis

of the 1982 manufacturing index. The manufacturing index used in this paper was obtained by splicing the two indexes at the year 1982 (see table 1). This index is referred to as the manufacturing index 1984. The index of manufacturing GDP largely approximates the manufacturing index 1984. Differences occur for three reasons.

Firstly, from 1978 there was concern that the 1971 manufacturing index underepresented electronic output. However, electronic output and employment data from 1978 suggest that the electronics sub-sector was not particularly significant before 1981.

Secondly, the nominal manufacturing GDP figures were revised in 1980 from 1978 onwards and there was some concern to revise the real GDP numbers as well. Unfortunately, errors were made and the real GDP numbers before 1978 remained unrevised.

Thirdly, the 1982 manufacturing index only became available in March 1985. Thus official reports such as the Government Economic Report 1984 will show GDP figures for the period 1982-84 based on the revised version of the 1971 manufacturing index, along with any errors made prior to 1978. For this reason, the index derived from the official GDP figures published prior to 1985 is difficult to substantiate.

Manufacturing Deflators

In addition to the manufacturing index 1984, four other reliable indexes may be calculated. These are a deflated sales

index, a deflated labour index, a deflated input index and a double inflation index, all for manufacturing.

The best procedure is to deflate each sub-category of manufacturing output (electronics, apparel etc.) by an appropriate wholesale price index. However, there are two basic difficulties. Firstly, no accurate series exist for the sub categories before 1974. Secondly, no wholesale price indices exist for the sub categories. Although a number of good proxies are available. The procedure used in this paper is to deflate the aggregate nominal manufacturing data i.e. sales, wages and inputs - using 1980 as the base year - by appropriate proxies. The nominal data was obtained from the Statistical Department. The official series covers the period 1974 - 1984. The unadjusted series for the period 1970-74 was adjusted for non response by making allowance for the difference in the values fo sales in 1974. [See Table 2]. Five deflators are applied to th output indicators (wages and sales) and the best ones chosen by statistical criteria (See Table 3). These are: the consumer price index, the manufacturing wages index, the total wages index, the manufacturing machinery price index and the manufacturing producer price index. The manufacturing machinery price index is a hybrid of the consumer price index and the US producer price index for machinery, while the manufacturing producer price index is a hybrid of the consumer price index and the US producer price index for industry. These hybrids contain the prices for the manufacturing items in the consumer price index and in addition contain the US producer prices which are

assumed to face enclave manufacturers, such as electronic firms in Barbados. The weights are determined by the weights for the manufacturing items in the CPI (food, beverages and tobacco, clothing and footwear) and by the ratio of exports to sales for manufacturing. The ratio of export to sales was 31% in test year 1980, and thus the weight applied to the US producer price indexes is 31%. This approach is plausible because the greater part of Barbados' trade and imported inflation arises in the U.S.

Correspondingly, the value of manufacturing imputs is deflated by the index for intermediate imports and the index for total imports (table 3). Indexes for sub-categories (such as manufacturing, wages and intermediate imports) and US machinery cover fewer items than indexes for total categories and are therefore likely to be more inaccurate. For this reason, indexes for the total categories are also used.

The double deflation indicator is obtained as the weighted average of the sales and input indicators. The weight is determined as the ratio of sales to value added in the base year 1980 - see appendix.

Reliability of Manufacturing Estimators

Econometric texts suggest two criteria for determining the best estimator - reliability and predictive power. It is usual to define reliability in terms of the relative sizes of the variances of unbiased estimators. However, it may be shown that

in some cases biassed estimators provide better estimates than unbiassed ones (Goldberger 1964). In this report, an estimator is defined to be more reliable if its second moment about the true parameter is the smaller of the two. The second moment of an estimator about a parameter (termed its mean square error) equals the sum of the variance of the estimator around its own expectation and the squared bias of the estimator around the true parameter. The true parameter is assumed to be the manufacturing index 1984 (previously defined). These concepts are clarified in the appendix. As noted earlier, five output deflators and two input deflators were developed. This will result in five sales estimators, five labour estimators, two input estimators, and ten double deflation estimators for which variances and mean square error will be calculated [Table 4]. However, it may be proved that the double deflation estimators are unbiased (see appendix). This is because the double deflation procedure utilises all available information on expenditures and prices. In contrast, input estimators ignore output information and vice versa.

Table 5 gives the variance, biased squared and mean square error for each estimator of the manufacturing index 1984. In general the labour estimators are more reliable as evidenced by their smaller mean square error. This results from both smaller variance and smaller bias (among the categories subject to bias). The most reliable estimator results from the deflation of labour payments by the consumer price index. Not a theoretically pleasing result. This indicates room for improvement of the manufacturing producer and wage indexes.

the smallest mean square error overall of 177.8. The manufacturing producer price index provided better results than the narrow manufacturing machinery price index. This is a satisfactory result since the machinery element of the manufacturing sector has only been significant over one third of the time series. Similarly, the use of the manufacturing wage deflator provides better results than the (total) wage deflator.

The output estimators performed reasonably well; with the deflation of sales by the consumer price index providing the best estimate. In general the input estimators were unreliable; the results of both high vaiance and high bias. However, as expected, the deflation of inputs by the intermediate import price index provided the better estimate.

The double deflation estimators did not perform well in general. However, the best double deflation estimator ranks fifth among the twenty-two estimators selected. The best double deflation estimator arises from the weighted average of the sales index deflated by the manufacturing producer price index and the input index deflated by the intermediate import price index. The best estimators overall are, in order of importance:

Estimator	Rank	Nominal Series	Deflator
L1	1	Labour Payments	Consumer Prices
L3	2	" "	MFG Producer Prices
L5	3	" "	MFG Wages
L2	4	" "	MFG Machinery Prices
D32	5	Sales (inputs)	MFG Producer Prices (Intermediate Import Prices)
01	6	Sales	Consumer Prices

Reliability of Expenditure Data Alone

Quantity relatives do not exist for most of the service sectors in Barbados. Thus, it is important to know whether it is feasible to determine the reliability of estimators using expenditure data (and deflators) alone. The procedure involves calculating the second moments of the estimators around the true double deflation parameter instead of the manufacturing index parameter (Hill 1971). The true double deflation index is formed by deducting the standard error from the double deflation index for each point in the series. The rationale for doing this is that it may be proved that if all data is accurate and consistent, the double deflation index will approximate the manufacturing index (See Appendix).

Table 6 describes the estimators of the true double deflation index. Theoretically, the twelve output and input estimators used in estimating the true double deflation index are also used in estimating the manufacturing index. However, in practice the estimators are defined differently because of the practical problem of estimating the bias squareds i.e. for each of the twelve estimators there are at least two biased squareds formed by combinators of output and input indices. For this reason there are effectively thirty estimators of the true double deflation parameter (Table 7).

The most obvious result is that the mean square errors of the estimators of the true double deflation estimators far

exceed most of the variances of the double deflation estimators. (compare table 7 with that of table 5). This implies that this method overwhelmingly favours the double deflation procedure. The best estimator (excluding values obtained from combinations) are in order of importance.

Estimator	Rank	Nominal Series	Deflator
D32	1	Sales (Inputs)	MFG Producer Prices (Intermediate Import Prices)
D22	2	Sales (Inputs)	MFG Machinery Prices (Intermediate Import Prices)
D12	3	Sales (Inputs)	Consumer Prices (Intermediate Import Prices)
O12	4	Sales	Consumer Prices
D42	5	Sales (Inputs)	Wage Index (Intermediate Import Prices)
O32	6	Sales	Manufacturing Producer Prices

The use of expenditure data alone provides different reliability results from the use of both expenditure data and quantity relatives. Thus for sectors for which quantity relatives do not exist, (i.e. service sectors) it will be impossible to determine reliability using statistical criteria alone. Some element of judgement is therefore required. The conflict in the two approaches is more clearly seen when the results are grouped by type of estimating procedure:

superior to other estimators and statistically significant at 5% level with R^2 of about 75-85%. The best result was obtained when labour payments were deflated by the consumer price indexes = scope for improvement of wage index. There may be some scope for the use of double deflation estimators if better deflators can be developed, or the data base improved. The best result was obtained using a weighted average of sales deflated by the manufacturing producer price-index and imputs deflated by the intermediate import price index; the weight is of course the ratio of sales to value added in the base year. Sales estimators did not perform well and inputs deflators provided the worst results. One important result is that it was impossible to establish satisfactory reliability criteria in the absence of quantity relatives. This indicates that for service sectors where quantity relatives largely do not exist, some element of judgement will be required.

Appendix

The Reliability of Alternative Real Product Estimators

The mathematical derivation of conclusions stated in the text (presented below) were first developed by (T.P. Hill).

The base weighted version of the index of real product or value added is given as follows: (for convenience the summation signs are omitted).

$$V = \frac{Poq_t - Poq_t^*}{Poq_o - Poq_o^*} \qquad (1)$$

where p and q refer to price and quantity respectively, while base year and current values are shown by o and t, respectively; the asterisk denotes that the price and quantity relate to intermediate input. The index of real product is derived by deflating current values by Laspeyres volume indexes for output and input denoted LL and LL*, respectively. It is usual to use paache indexes but none are available in Barbados. Laspeyres indexes may be used if applied at a fairly low level of aggregation to facilitate manipulation of the summations.

$$V = \frac{\frac{Ptq_t}{LL} - \frac{Ptq_t^*}{LL^*}}{Poq_o - Poq_o^*} \qquad (2)$$

Defining A as the ratio of output to value added in the base year it can be shown that

$$V = AZ - (A-1)G. \qquad (3)$$

Appendix Cont'd

where $A = \frac{Poq_o}{Poq_o - Poq_o^*}$ 1 (4)

$Z = \frac{Poq_t}{Poq_o}$ = Laspeyres output volume index (5)

$G = \frac{Poq_t^*}{Poq_o^*}$ = Laspeyres input volume index (6)

V, Z and G refer to the true values of the double deflation output and input indices. In practice the indices are measured with some error as follows:

$$O = Z + e_1 \qquad (7)$$

$$I = G + e_2$$

$$D = AO - (A-1)\ I$$

where e_1 and e_2 are independent random errors of measurement with expected values of zero. However, the output value added coefficient A is assumed to measured without error. Three ways of estimating value added are compared:

(a) Double Deflation

The double deflation estimate of value added denoted D, is given as follows:

$$D = V + Ae_1 - (A - 1)\ e_2 \qquad (8)$$

Since the expected values of the errors are zero, E(D) = V so the double deflation estimate is unbiased.

(B) Output As A Single Indicator

It can be shown that the output estimate of valued added is simply O, expressed as follows:

$$O = V - (A - 1)(Z - G) + e_1 \quad (9)$$

Taking expected values, evidently O is a biased estimator of V with the bias equal to $(A - 1)(Z - G)$.

(c) Input As A Single Indicator

It can be shown that the input estimate of value added is simply I, expressed as follows:

$$I = V - A(Z - G) + e_2 \quad (10)$$

Taking expected values evidently I is a biased estimator of V with the bias equal to $A(Z - G)$.

(d) Relative Efficiency of the Estimates

The best estimate is obtained by examining the dispersion of each estimate about V as given by their second moments. It follows from equations 8, 9 and 10 that the best estimates will be obtained by examining the following equations:

$$E(D - V)^2 = (A_{e1})^2 - (A - 1)^2 (e_2)^2 \quad (11)$$

$$E(O - V)^2 = (A - 1)^2 (Z - G)^2 + (e_1)^2 \quad (12)$$

Mean Square Error (L) $= E(L - V)^2$

Bias Squared $= (E(L) - V)^2$

Variance (L) $= E(L - E(L)2$

$$E(I - V)^2 = A^2 (Z - G)^2 + (e_2)^2 \quad (13)$$

This procedure is also referred to as the mean square error.

(c) Expenditure Reliability Criteria

Expenditure reliability criteria are obtained by examining the dispersion of each estimate about the 'true' double deflation index D*, expressed as follows:

$$D = D^* + ed$$

Where ed is the standard error of d. The reliability of the true double deflation index is thus obtained by examining the following equations:

$$E(D-D^*)^2 = (ed)^2$$

$$E(O-D^*)^2 = (z-o^*)^2 + (e_1)^2 \quad (14)$$

$$E(I-D^*)^2 = (G-D^*)^2 + e_2)^2 \quad (15)$$

$$D^* = E(D) = ed$$

Table 1

Indexes of Real MFG Product 1980 = 100

Year	MFG 1984	MFG 1971	* MFG 1982	MGDP
1970	62.15	62.15	n.a.	55.06
1971	67.80	67.80	"	58.57
1972	75.19	75.19	"	64.65
1973	79.80	79.80	"	69.31
1974	76.07	76.07	"	66.07
1975	82.64	82.64	"	73.07
1976	97.42	97.42	"	85.43
1977	98.37	98.37	"	88.14
1978	100.14	100.14	"	96.20
1979	97.83	97.82	"	97.85
1980	100.00	100.00	"	100.00
1981	94.51	94.51	105.7	96.40
1982	89.41	89.63	100.0	96.79
1983	91.73	88.33	102.6	98.62
1984	93.43	81.22	104.5	95.17

* 1982 = 100

Source: Central Bank of Barbados
Barbados Statistical Department

Table 2

Nominal Manufacturing GDP 1970-1984

(Producer Values)

$ Million

Year	Unadjusted Sales	Official Sales	Adjusted Accounts		
			Sales	Inputs	Labour Payments
1970	95.8		116.6	83.1	19.0
1971	108.5		132.0	91.8	22.5
1972	125.3		152.5	101.9	27.1
1973	153.8		187.1	125.9	26.3
1974	197.7	240.7	240.7	178.0	39.2
1975		275.3	275.3	203.4	42.9
1976		304.3	304.3	219.5	55.7
1977		339.5	339.5	236.9	65.4
1978		391.9	391.9	279.5	71.8
1979		468.9	468.9	332.5	69.0
1980		587.5	587.5	418.5	86.0
1981(P)		683.4	683.4	493.7	99.8
1982(P)		705.4	705.4	499.9	103.0
1983(P)		810.7	810.7	572.0	118.4
1984(P)		913.2	913.2	649.1	133.3

(P) denotes provisional

Source: Central Bank of Barbados
Barbados Statistical Service

Table 3

Manufacturing Deflators 1980=100

Year	Consumer Price Index	Wages Index	MFG Wage Index	MFG Machinery Price Index	MFG Producer Price Index	Import Price Index	Import Intermediate Price Index
1970	27.3	31.6	34.2	31.7	29.7	34.4	27.3
1971	30.7	36.5	34.3	34.6	32.5	36.6	30.8
1972	32.8	40.1	38.1	36.4	34.5	39.3	32.5
1973	38.4	43.5	38.9	40.4	38.9	47.5	36.5
1974	53.3	49.3	44.1	51.9	51.2	64.5	66.1
1975	64.1	54.3	51.4	63.1	61.5	70.7	71.9
1976	67.3	62.2	67.7	66.7	65.2	72.2	71.0
1977	73.0	70.8	74.4	71.5	70.1	77.3	72.2
1978	79.9	78.5	79.0	77.1	75.4	95.0	78.0
1979	90.4	84.2	89.0	85.1	84.2	95.0	89.3
1980	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1981	118.6	109.8	113.8	113.4	110.0	106.0	101.2
1982	130.8	121.6	129.1	122.4 (P)	121.1(P)	107.4	105.7
1983	137.7	128.3	140.4	128.5(P)	127.2(P)	110.2	108.3
1984	144.1	134.7(P)	146.9(P)	132.0(P)	130.6(P)	115.0(P)	112.8(P)

(P) denotes provisional

Source: Central Bank of Barbados
USDOC Survey of Current Business

Table 4

Description of Manufacturing Estimators

Manufacturing Estimator	Description	Deflator
L1	Deflated Labour Index	Consumer Price Index
L2	" "	Manufacturing Machinery Price Index
L3	" "	Manufacturing Producer Price Index
L4	" "	Wages Index
L5	" "	Manufacturing Wages Index
01	Deflated Sales Index	Consumer Price Index
02	" "	Manufacturing Machinery Price Index
03	" "	Manufacturing Producer Price Index
04	" "	Wages Index
05	" "	Manufacturing Wages Index
I1	Deflated Input Index	Import Price Index
I2	" "	Intermediate Import Price Index

Table 4 Cont'd

Description of Manufacturing Estimators

Manufacturing Estimator	Description	Output Index	Index Index
D11	Double Deflation Index	01	I1
D21	" "	02	I1
D31	" "	03	I1
D41	" "	04	I1
D51	" "	05	I1
D12	" "	01	I2
D22	" "	02	I2
D32	" "	03	I2
D42	" "	04	I2
D52	" "	05	I2

Table 5

Reliability of Estimators of the Manufacturing Index 1984

Estimator	Variance	Bias Squared	Mean Square Error
L1	82.0	95.8	177.8
L2	200.0	99.8	299.8
L3	176.9	67.2	244.1
L4	224.7	161.5	386.2
L5	150.5	116.9	267.4
O1	144.1	226.9	371.0
O2	263.1	287.0	550.1
O3	237.1	202.1	439.2
O4	281.2	325.6	606.8
O5	202.8	339.3	542.1
I1	650.6	804.0	1,454.6
I2	529.6	461.8	991.4
D11	1,282.1	0.0	1,282.1
D21	382.0	0.0	382.0
D31	374.7	0.0	374.7
D41	570.5	0.0	570.5
D51	1,423.7	0.0	1,423.7
D12	701.3	0.0	701.3
D22	469.0	0.0	469.0
D32	353.1	0.0	353.1
D42	1,021.5	0.0	1,021.5
D52	1,702.8	0.0	1,702.8

Table 6

Description of Double Deflation Estimators

Manufacturing Estimator	Description	Output Index	Index Index
L11	Deflated Labour Index	G11	I1
L21	" "	G21	I1
L31	" "	G31	I1
L41	" "	G41	I1
L51	" "	G51	I1
L12	" "	G12	I2
L22	" "	G22	I2
L32	" "	G32	I2
L42	" "	G42	I2
L52	" "	G52	I2
O11	Deflated Sales Index	O1	I1
O21	" "	O2	I1
O31	" "	O3	I1
O41	" "	O4	I1
O51	" "	O5	I1
O12	" "	O1	I2
O22	" "	O2	I2
O32	" "	O3	I2
O42	" "	O4	I2
O52	" "	O5	I2

Table 6 Cont'd

Description of Double Deflation Estimators

Manufacturing Estimator	Description	Output Index	Index Index
I11	Deflated Input Index	O1	I1
I21	" "	O2	I1
I31	" "	O3	I1
I41	" "	O4	I1
I51	" "	O5	I1
I12	" "	O1	I2
I22	" "	O2	I2
I32	" "	O3	I2
I42	" "	O4	I2
I52	" "	O5	I2

Table 7

Reliability of Estimators of the True Double Deflation Index (Double Deflation Estimators Excluded)

Estimator	Variance	Bias Squared	Mean Square Error
L11	82.0	5,572.2	5,654.2
L12	200.0	3,208.4	3,408.4
L31	176.9	4,115.0	4,291.4
L41	224.7	3,409.2	3,633.4
L51	150.5	3,405.6	4,056.1
L12	82.0	2,681.4	2,763.4
L22	200.0	2,250.5	2,450.5
L32	176.9	2,604.5	2,781.4
L42	224.7	3,374.0	3,598.7
L52	150.5	3,395.7	3,546.2
011	144.1	2,257.3	2,401.4
021	263.1	1,252.6	1,515.7
031	237.1	1,641.6	1,928.7
041	281.2	1,531.5	1,812.7
051	202.8	1,983.3	2,186.1
012	144.1	661.1	805.2
022	263.1	1,252.6	1,515.7
032	237.1	805.0	1,042.1
042	281.2	2,118.2	2,399.4
052	202.8	2,035.5	2,238.3
I11	650.6	4,448.4	5,099.0

Table 7 Cont'd

Reliability of Estimators of the True Double Deflation Index (Double Deflation Estimators Excluded)

Estimator	Variance	Bias Squared	Mean Square Error
I21	650.6	2,468.3	3,118.4
I31	650.6	3,333.5	3,984.1
I41	650.6	4,394.5	5,045.1
I51	650.6	3,908.4	4,554.0
I12	529.6	1,302.8	1,832.4
I22	529.6	1,704.7	2,244.3
I32	529.6	1,586.6	2,116.2
I42	529.6	4,174.2	4,703.8
I52	529.6	4,011.3	4,540.9

Table 8

Current Real Product Estimation Procedures in Barbados

Activity	Output Indicator	Input Indicator	Price Indicator	Value Added Indicator	Judgement
Tourism	Adjusted Tourist Days				
Sugar	Tonnage				
Fishing	Tonnage				
Other					
Agriculture	Tonnage		WPI	NOM GDP	
Elec. & Gas	Volume, KWH				
Manufact.				IIP	
Constr.		Imports	CPI	NOM GDP	
Govt.	Employment Wages		 WI		
Wholesale	Tourism) Sugar) Mfg.)	Imports	CPI	NOM GDP	
Finance			CPI	NOM GDP	
Business	Wholesale	Imports	CPI	NOM GDP	
Mining Quarrying			IIP IIP		

Note: WPI = Wholesale price index for activity

WI = Wages index

CPI = Consumer price index for activity or average

IIP = Index of industrial production for activity

NOM GDP = Nominal value added of activity